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Edexcel A level
PHYSICS
2

Miles Hudson

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Contents

TOPIC 6 Further mechanics

TOPIC 7 Electric and magnetic fields

TOPIC 8 Nuclear and particle physics

TOPIC 9 Thermodynamics

TOPIC 10 Nuclear radiation

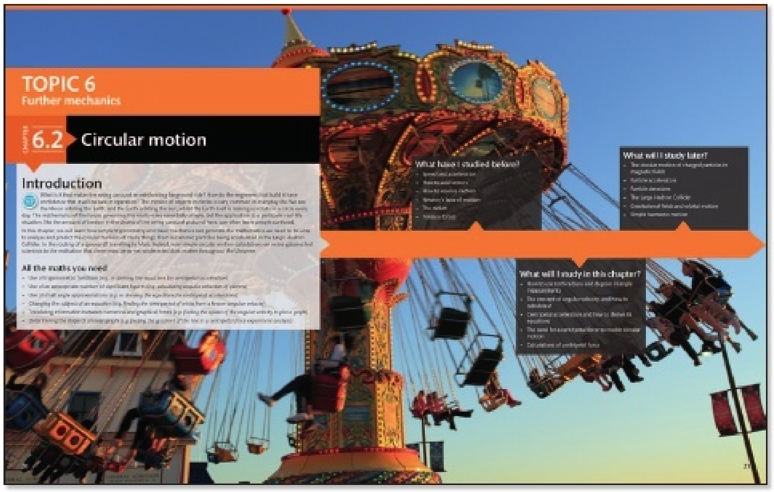
TOPIC 11 Gravitational fields

TOPIC 12 Space

TOPIC 13 Oscillations

How to use this book

Welcome to Book 2 of your A level Physics course. This book, covering Topics 6–13 of the specification, follows on from Book 1, which covered Topics 1–5. The exams that you will sit at the end of your A level course will cover content from both books. The following features are included to support your learning:



TOPIC 6
Further mechanics

6.2 Circular motion

Introduction

All the maths you need

What have I studied before?

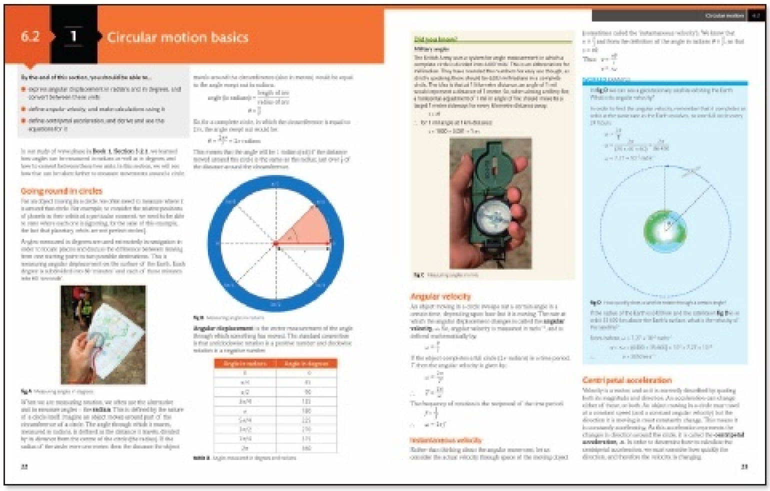
What will I study later?

What will I study in this chapter?

Chapter openers

Each chapter starts by setting the context for that chapter's learning:

- Links to other areas of physics are shown, including previous knowledge that is built on in the chapter, and future learning that you will cover later in your course.
- The **All the maths you need** checklist helps you to know what maths skills will be required.



6.2 1 Circular motion basics

Main content

The main part of each chapter covers all the points from the specification that you need to learn. The text is supported by diagrams and photos that will help you understand the concepts.

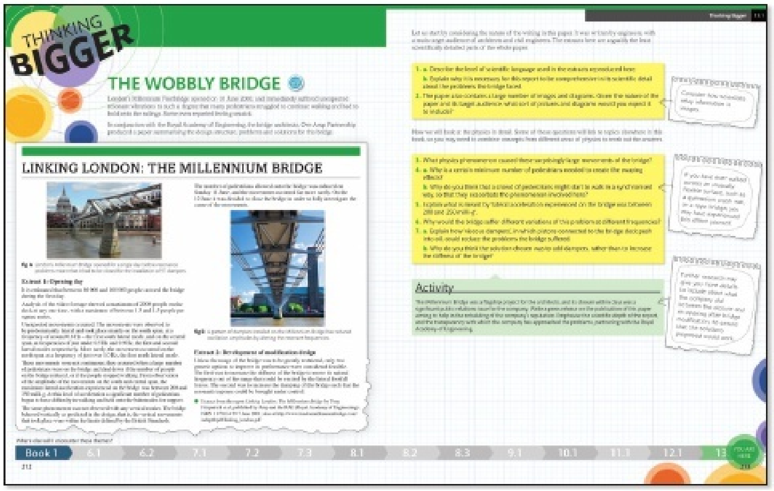
Within each section, you will find the following features:

- **Learning objectives** at the beginning of each section, highlighting what you need to know and understand.
- **Key definitions** shown in bold and collated at the end of each section for easy reference.
- **Worked examples** showing you how to work through questions, and how your calculations should be set out.
- **Investigations** provide a summary of practical experiments that explore key concepts. This includes the core practicals that you need to complete as part of your course and which may be assessed in your examinations. You can find detailed worksheets for the core practicals on the Edexcel website.
- **Learning tips** to help you focus your learning and avoid common errors.
- **Did you know?** boxes featuring interesting facts to help you remember the key concepts.
- **Working as a Physicist** icons highlight key sections that develop your skills as a scientist and relate to the Working as a Physicist section of the specification. WP
- **Questions** to help you check whether you have understood what you have just read, and whether there is anything that you need to look at again. Answers to the questions can be found on Pearson's website as a free resource.

Thinking Bigger

At the end of each chapter there is an opportunity to read and work with real-life research and writing about science. The timeline at the bottom of the spreads highlights which other chapters the material relates to. These spreads will help you to:

- read real-life material that's relevant to your course
- analyse how scientists write
- think critically and consider the issues
- develop your own writing
- understand how different aspects of your learning piece together.



THINKING BIGGER

THE WOBBLY BRIDGE

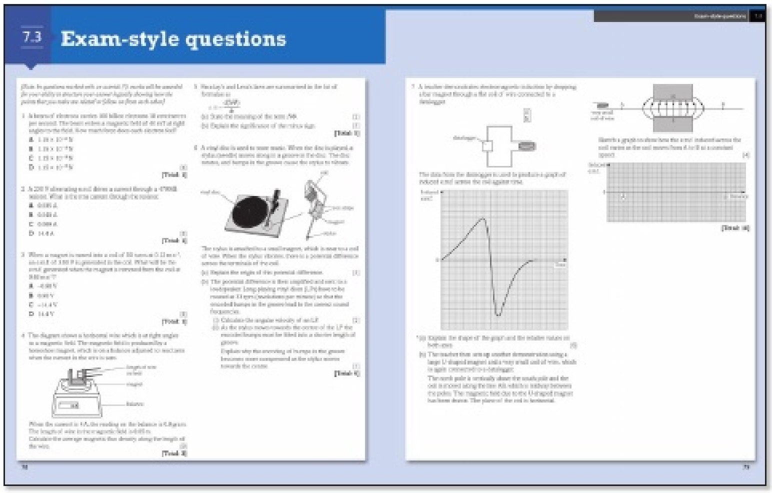
LINKING LONDON: THE MILLENNIUM BRIDGE

Activity

Exam-style questions

At the end of each chapter there are also **Exam-style questions** to help you to:

- test how fully you have understood the learning
- practise for your exams.



7.3 Exam-style questions

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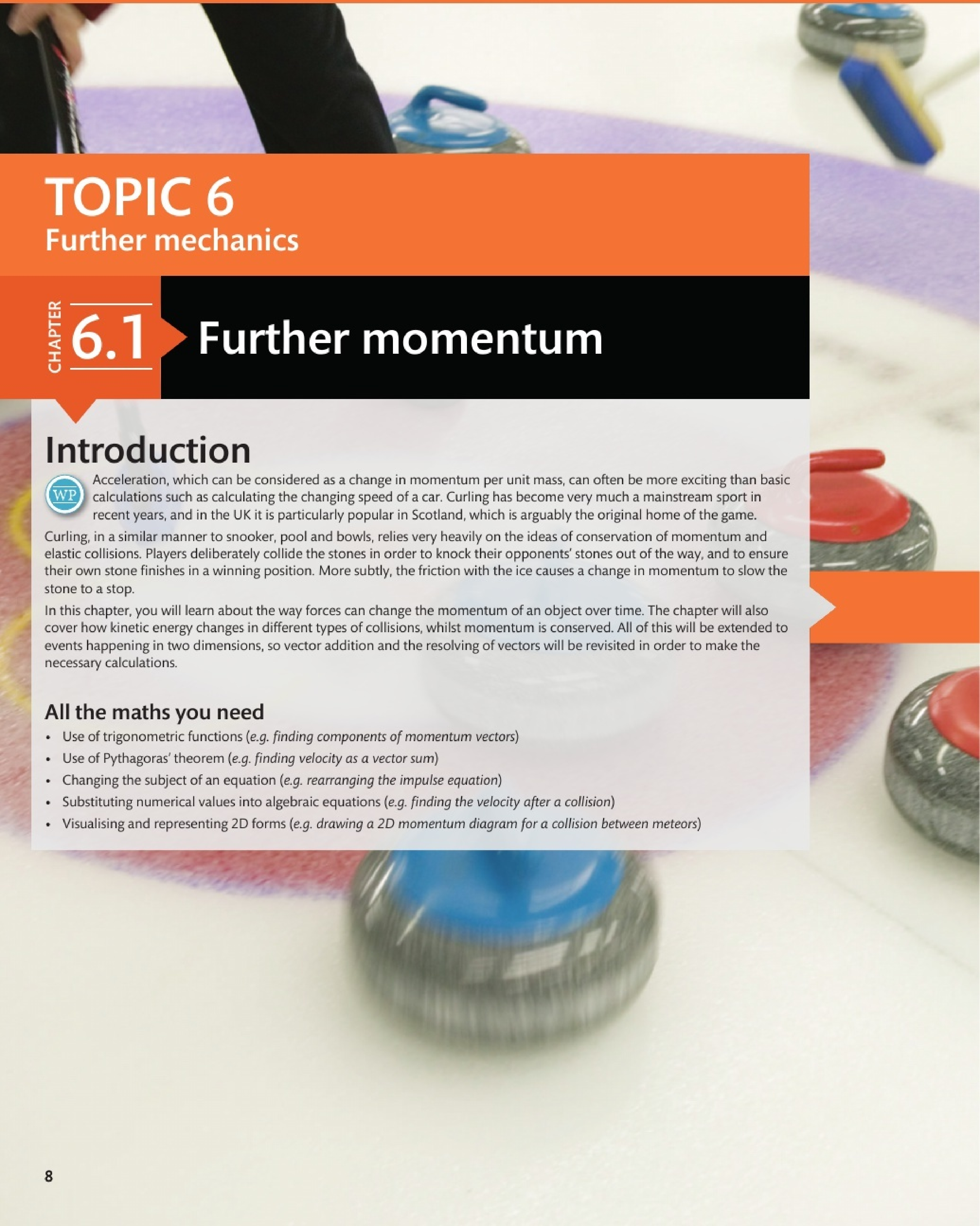


TOPIC 6
Further mechanics

CHAPTER 6.1 Further momentum

Introduction

Acceleration, which can be considered as a change in momentum per unit mass, can often be more exciting than basic calculations such as calculating the changing speed of a car. Curling has become very much a mainstream sport in recent years, and in the UK it is particularly popular in Scotland, which is arguably the original home of the game.

Curling, in a similar manner to snooker, pool and bowls, relies very heavily on the ideas of conservation of momentum and elastic collisions. Players deliberately collide the stones in order to knock their opponents' stones out of the way, and to ensure their own stone finishes in a winning position. More subtly, the friction with the ice causes a change in momentum to slow the stone to a stop.

In this chapter, you will learn about the way forces can change the momentum of an object over time. The chapter will also cover how kinetic energy changes in different types of collisions, whilst momentum is conserved. All of this will be extended to events happening in two dimensions, so vector addition and the resolving of vectors will be revisited in order to make the necessary calculations.

All the maths you need

- Use of trigonometric functions (*e.g. finding components of momentum vectors*)
- Use of Pythagoras' theorem (*e.g. finding velocity as a vector sum*)
- Changing the subject of an equation (*e.g. rearranging the impulse equation*)
- Substituting numerical values into algebraic equations (*e.g. finding the velocity after a collision*)
- Visualising and representing 2D forms (*e.g. drawing a 2D momentum diagram for a collision between meteors*)

What have I studied before?

- How to add forces as vectors
- How to resolve vectors
- Newton's laws of motion
- How to calculate kinetic energy
- Conservation of energy
- How to calculate the momentum of an object
- Conservation of linear momentum in collisions

What will I study in this chapter?

- The impulse equation and its connection with Newton's second law of motion
- The relationship between the force on an object and its change in momentum
- Conservation of linear momentum in two dimensions
- How to analyse collisions in 2D
- The difference between an elastic and an inelastic collision
- The equation for the kinetic energy of a non-relativistic particle, in terms of its momentum

What will I study later?

- How electrical and magnetic fields affect the momentum of charged particles
- How the momentum affects the size of a circle in which a charged particle is trapped by a magnetic field
- The de Broglie wavelength for a particle and its connection with the momentum of the particle
- Large-angle alpha particle scattering indicating the structure of the atom, with the scattering dependent on momentum conservation
- How conservation of momentum affects the creation and detection of new particles
- How ideas of momentum conservation in gas-particle collisions lead to equations for use in the kinetic theory of gases

6.1 1 Energy in collisions

By the end of this section, you should be able to...

- explain the difference between elastic and inelastic collisions
- make calculations based on the conservation of linear momentum to determine energy changes in collisions
- derive and use the equation for the kinetic energy of a non-relativistic particle

We have seen in **Book 1, Chapter 2.3** that linear momentum is always conserved in any collision between objects, and this is responsible for Newton's third law of motion. We also learned that Newton's second law of motion expresses the concept that a force is equivalent to the rate of change of momentum. **Book 1, Chapter 2.1** explained how forces can do work, as the means by which energy is transferred. So, does the kinetic energy change in a collision?

fig A Damaging a car uses energy, so what can we say about the conservation of kinetic energy in a car crash?

Elastic collisions

In a collision between one snooker ball and another, often the first one stops dead and the second then moves away from the collision. As both snooker balls have the same mass, the principle of conservation of momentum tells us that the velocity of the second ball must be identical to the initial velocity of the first. This means that the kinetic energy of this system of two balls before and after the collision must be the same. A collision in which kinetic energy is conserved is called an **elastic collision**. In general, these are rare. A Newton's cradle is an example that is nearly perfectly elastic (a tiny amount of energy is lost as sound). A collision caused by non-contact forces, such as alpha particles being scattered by a nucleus (see **Section 10.1.1**), is perfectly elastic.

fig B Newton's cradle maintains kinetic energy, as well as conserving momentum in its collisions.

Inelastic collisions

In a crash between two dodgems at the fair, the total momentum after the collision must be identical to the total momentum prior to it. However, if we calculate the total kinetic energy before and after, we invariably find that the total is reduced by the collision. Some of the kinetic energy is transferred into other forms such as heat and sound. A collision in which total kinetic energy is not conserved is called an **inelastic collision**.

Inelastic collision example

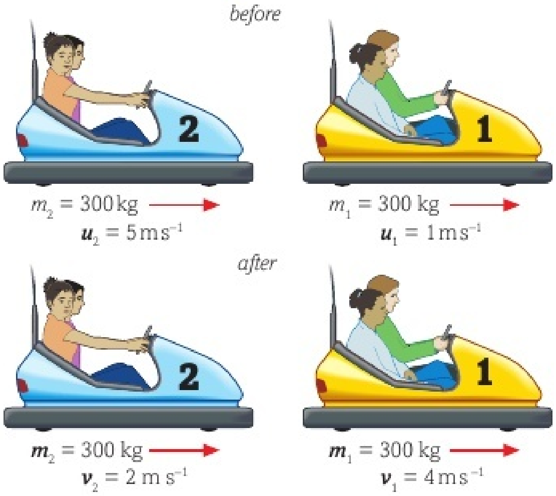


fig C The fun of inelastic collisions.

If you calculate the total momentum before and after the collision in **fig C**, you will see that it is conserved. However, what happens to the kinetic energy?

Before collision:

$$E_{k1} = \frac{1}{2} m_1 \boldsymbol{u}_1{}^2 = \frac{1}{2} \times (300) \times 1^2 = 150 \text{ J}$$

$$E_{k2} = \frac{1}{2} m_2 \boldsymbol{u}_2{}^2 = \frac{1}{2} \times (300) \times 5^2 = 3750 \text{ J}$$

Total kinetic energy = 3900 J

After collision:

$$E_{k1} = \frac{1}{2} m_1 \boldsymbol{v}_1{}^2 = \frac{1}{2} \times (300) \times 4^2 = 2400 \text{ J}$$

$$E_{k2} = \frac{1}{2} m_2 \boldsymbol{v}_2{}^2 = \frac{1}{2} \times (300) \times 2^2 = 600 \text{ J}$$

Total kinetic energy = 3000 J

Loss in kinetic energy = 900 J. This is an inelastic collision.

This 'lost' energy will have been transferred into heat and sound energy.

Learning tip

Remember that the idea of energy conservation, when considered in order to decide whether a collision is elastic or inelastic, only relates to the *kinetic* energy. Total energy in all forms must always be conserved.

Investigation

Investigating elastic and inelastic collisions

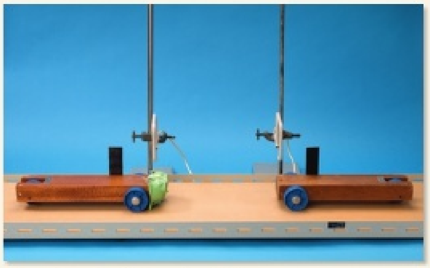


fig D Crash testing the elasticity of collisions.

You can investigate elastic and inelastic collisions in the school laboratory. If you cause head-on collisions, and record the mass and velocity of each trolley before and after the collisions, you then calculate the momentum at each stage. This should be conserved. You can also then calculate kinetic energy before and after the collisions. Real cars are designed to absorb as much kinetic energy as possible when they crash. This reduces the energy available to cause injury to the passengers. What is the best design on your experimental trolleys for a crumple zone to fulfil this idea of kinetic energy absorption?

Particle momentum

We know that the formula for calculating kinetic energy is $E_k = \frac{1}{2}mv^2$ and that the formula for momentum is $p = mv$. By combining these we can get an equation that gives kinetic energy in terms of the momentum and mass.

$$E_k = \frac{1}{2}m\boldsymbol{v}^2 \quad \text{and} \quad \boldsymbol{v} = \frac{\boldsymbol{p}}{m}$$

$$E_k = \frac{1}{2}m\boldsymbol{v}^2$$

$$\therefore \quad E_k = \frac{1}{2}m\left(\frac{\boldsymbol{p}}{m}\right)^2$$

$$\therefore \quad E_k = \frac{1}{2}\frac{\boldsymbol{p}^2}{m}$$

$$\therefore \quad E_k = \frac{\boldsymbol{p}^2}{2m}$$

This formula is particularly useful for dealing with the kinetic energy of subatomic particles travelling at non-relativistic speeds – that is, much slower than the speed of light.

Particle collisions

In experiments to determine the nature of fundamental particles, physicists detect the movements of many unknown particles. The Large Hadron Collider experiment at CERN, underground near Geneva in Switzerland, produces 600 million particle interactions in its detector every second. The conservation of momentum allows the mass of these particles to be calculated, which helps towards their identification. This can be done by colliding the particles produced in the experiment with known particles in the detector.

For example, if the detector registers an elastic collision with one of its neutrons, changing the neutron's velocity from stationary to $3.4 \times 10^6\,\text{m}\,\text{s}^{-1}$ in a head-on collision with an unknown particle, which was initially moving at 10% of the speed of light, and leaves the collision in the opposite direction at $1.09 \times 10^3\,\text{m}\,\text{s}^{-1}$, what is the mass of the mystery particle? The mass of a neutron is 1.67×10^{-27} kg.

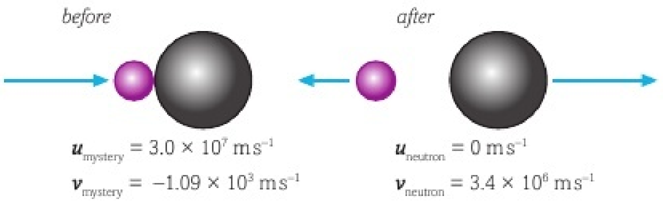


fig E Discovering mystery particles from their momentum and collisions.

Before collision:

$$\boldsymbol{p}_{\text{mystery}} = m_{\text{mystery}} \times 3 \times 10^7 = \boldsymbol{p}_{\text{total before}} \qquad (\boldsymbol{p}_{\text{n}} = \text{zero})$$

After collision:

$$\begin{aligned}\boldsymbol{p}_{\text{total after}} &= (m_{\text{mystery}} \times \boldsymbol{v}_{\text{mystery}}) + (m_{\text{n}} \times \boldsymbol{v}_{\text{n}})\\ &= (m_{\text{mystery}} \times -1.09 \times 10^3) + (1.67 \times 10^{-27} \times 3.4 \times 10^6)\\ &= \boldsymbol{p}_{\text{total before}} = m_{\text{mystery}} \times 3 \times 10^7\end{aligned}$$

Thus:

$$(m_{\text{mystery}} \times -1.09 \times 10^3) + (1.67 \times 10^{-27} \times 3.4 \times 10^6) = m_{\text{mystery}} \times 3 \times 10^7$$

$$(1.67 \times 10^{-27} \times 3.4 \times 10^6) = (m_{\text{mystery}} \times 3 \times 10^7) - (m_{\text{mystery}} \times -1.09 \times 10^3)$$

$$5.678 \times 10^{-21} = (m_{\text{mystery}} \times 30\,001\,090)$$

So:

$$\begin{aligned}m_{\text{mystery}} &= \frac{5.678 \times 10^{-21}}{30\,001\,090}\\ &= 1.89 \times 10^{-28}\,\text{kg}\end{aligned}$$

This is approximately 207 times the mass of an electron, and so this can be identified as a particle called a muon, which is known to have this mass.

Questions

1 An alpha particle consists of two protons and two neutrons. Calculate the kinetic energy of an alpha particle which has a momentum of $1.08 \times 10^{-19}\,\text{kg}\,\text{m}\,\text{s}^{-1}$:
(a) in joules
(b) in electron volts
(c) in MeV.
(mass of neutron = mass of proton = 1.67×10^{-27} kg)

2 A bowling ball travelling at $5\,\text{m}\,\text{s}^{-1}$ strikes the only standing pin straight on. The pin flies backward at $7\,\text{m}\,\text{s}^{-1}$. Calculate:
(a) the velocity of the bowling ball after the collision
(b) the loss of kinetic energy in this collision.
(mass of bowling ball = 6.35 kg; mass of pin = 1 kg)

3 In a particle collision experiment, a mystery particle was detected to have collided with a stationary neutron and set the neutron into motion with a velocity of $1.5 \times 10^7\,\text{m}\,\text{s}^{-1}$. If the mystery particle arrived at a velocity of 1% of the speed of light, and recoiled after collision with a velocity of $7.5 \times 10^5\,\text{m}\,\text{s}^{-1}$ in the opposite direction, calculate the mass of the mystery particle, and so identify it.

Key definitions

An **elastic collision** is a collision in which total kinetic energy is conserved.

An **inelastic collision** is a collision in which total kinetic energy is not conserved.

6.1 2 More collisions

By the end of this section, you should be able to...

- apply the conservation of linear momentum to situations in two dimensions
- analyse collisions in two dimensions
- calculate impulses and changes in momentum

We have so far only considered the conservation of linear momentum in one-dimensional collisions, where all objects move forwards and/or backwards along the *same* straight line. This is an unusual situation, and we need to be able to work with more complex movements. Helpfully, momentum is conserved in each dimension separately. So, we resolve vector movements entering a collision into components in each dimension and then calculate following the conservation of momentum in each dimension. After this, we can recombine component vectors to give us an overall vector after a collision. A real world example, as in **fig A**, will illustrate this best.

Investigation

Investigating impulse

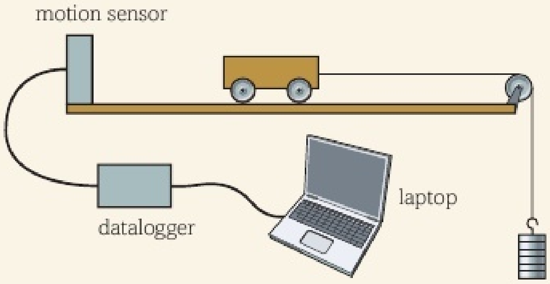


fig A Measuring how impulse changes the momentum of a trolley

In **Book 1, Section 2.3.1** we saw how you can investigate the change in momentum over time for a trolley that is subject to a constant accelerating force. Using the same apparatus, you could again record how different forces acting over different time periods cause the trolley to accelerate to different velocities. From these results, you can calculate the impulse applied in each case. As $\boldsymbol{I} = \boldsymbol{F}\Delta \boldsymbol{t} = \Delta\boldsymbol{p} = \Delta m\boldsymbol{v}$, a graph of impulse on the y-axis against change in velocity on the x-axis should give a straight best fit line through the origin. This straight line verifies the impulse equation, and the gradient of it will give the mass of the accelerating trolley and weights. Make sure you have a good understanding of this practical as your understanding of the experimental method may be assessed in your examinations.

Impulse

The product of a force applied for a certain time ($\boldsymbol{F} \times \Delta t$) is known as the **impulse**, and this is equal to the change in momentum:

$$\begin{aligned} \text{impulse (Ns)} &= \text{force (N)} \times \text{time (s)} \\ &= \text{change in momentum (kg m s}^{-1}\text{)} \\ \text{impulse} &= \boldsymbol{F} \times \Delta t = \Delta \boldsymbol{p} \end{aligned}$$

To stop something moving, we need to remove all of its momentum. This idea allows us to calculate the impulse needed to stop an object moving. If we know how long a force is applied, we could work out the size of that force.

WORKED EXAMPLE

What is the impulse needed to accelerate a 1000 kg car from rest to 25 m s^{-1}?

$$\boldsymbol{p} = m \times \boldsymbol{v}$$

At the start, the car is at rest, so has no momentum. Therefore the change in momentum will equal its final momentum:

$$\begin{aligned} \Delta \boldsymbol{p} &= 1000 \times 25 \\ \text{impulse} &= \boldsymbol{F} \times \Delta t = \Delta \boldsymbol{p} \\ \boldsymbol{I} &= 25\,000 \text{ kg m s}^{-1} \end{aligned}$$

If the car needed to stop in 3.8 seconds, what force would the brakes need to apply?

At the end, the car is at rest, so has no momentum. Therefore the change in momentum will equal its initial momentum:

$$\begin{aligned} \boldsymbol{F}t &= m\Delta \boldsymbol{v} \\ m\Delta \boldsymbol{v} &= 25\,000 \text{ kg m s}^{-1} \\ \boldsymbol{F} &= \frac{m\Delta \boldsymbol{v}}{t} \\ \boldsymbol{F} &= \frac{25\,000}{3.8} \\ \boldsymbol{F} &= 6600 \text{ N to 2 significant figures} \end{aligned}$$

Collision vectors

A spacecraft is moving through empty space at 8 m s^{-1}. A meteoroid, travelling at 15 m s^{-1}, comes from behind and at an angle of 45° to the line of movement of the rocket, crashes into the rocket and becomes embedded in it. The rocket has a mass of 350 kg and the meteorite mass is 20 kg. Calculate the velocity of the rocket after the collision.

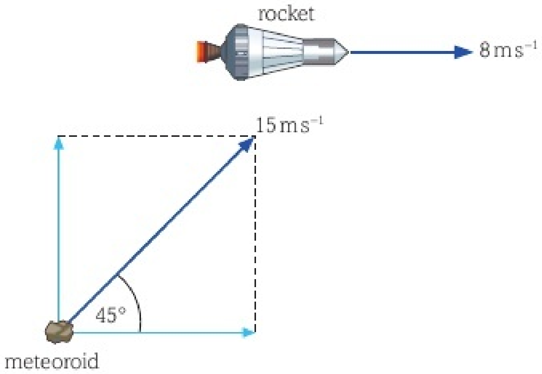


fig B A collision in two dimensions.

Before collision:

Parallel to rocket motion:

$$\begin{aligned} \boldsymbol{v}_{\text{meteorite}} &= 15\cos 45° = 10.6\,\text{m s}^{-1} \\ \boldsymbol{p}_{\text{meteorite}} &= 20 \times 10.6 = 212\,\text{kg m s}^{-1} \\ \boldsymbol{p}_{\text{rocket}} &= 350 \times 8 = 2800\,\text{kg m s}^{-1} \\ \boldsymbol{p}_{\text{parallel}} &= 2800 + 212 = 3012\,\text{kg m s}^{-1} \end{aligned}$$

Perpendicular to rocket motion:

$$\begin{aligned} \boldsymbol{v}_{\text{meteorite}} &= 15\sin 45° = 10.6\,\text{m s}^{-1} \\ \boldsymbol{p}_{\text{meteorite}} &= 20 \times 10.6 = 212\,\text{kg m s}^{-1} \\ \boldsymbol{p}_{\text{rocket}} &= 350 \times 0 = 0 \\ \boldsymbol{p}_{\text{perpendicular}} &= 0 + 212 = 212\,\text{kg m s}^{-1} \end{aligned}$$

After collision:

Vector sum of momenta (**fig C**):

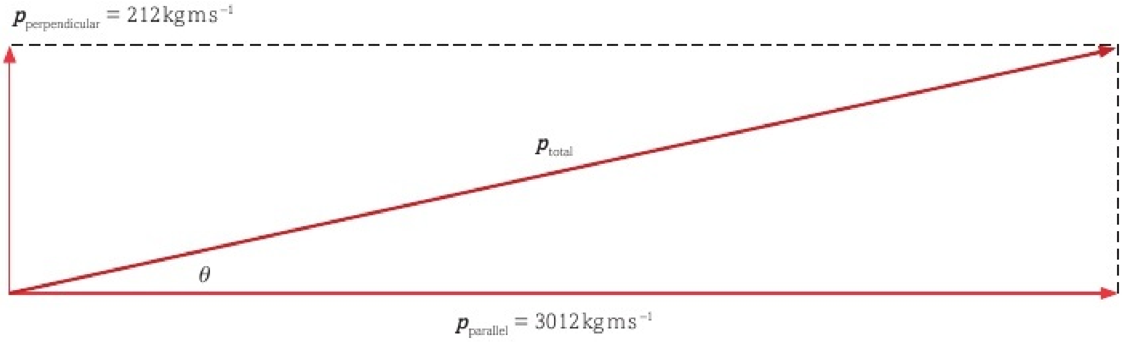


fig C Vector sum of total momenta in two dimensions.

$$\boldsymbol{p}_{\text{total}} = \sqrt{(3012^2 + 212^2)} = 3019\,\text{kg m s}^{-1}$$

$$\boldsymbol{v}_{\text{after}} = \frac{\boldsymbol{p}_{\text{total}}}{(m_{\text{rocket}} + m_{\text{meteorite}})} = \frac{3019}{(350 + 20)} = 8.16\,\text{m s}^{-1}$$

Angle of momentum (i.e. direction of velocity) after collision:

$$\theta = \tan^{-1}\left(\frac{212}{3012}\right) = 4.0°$$

So, the spacecraft with embedded meteorite carries on at 8.16 m s^{-1} at an angle of 4.0° off the original direction of motion.

Deep space collision

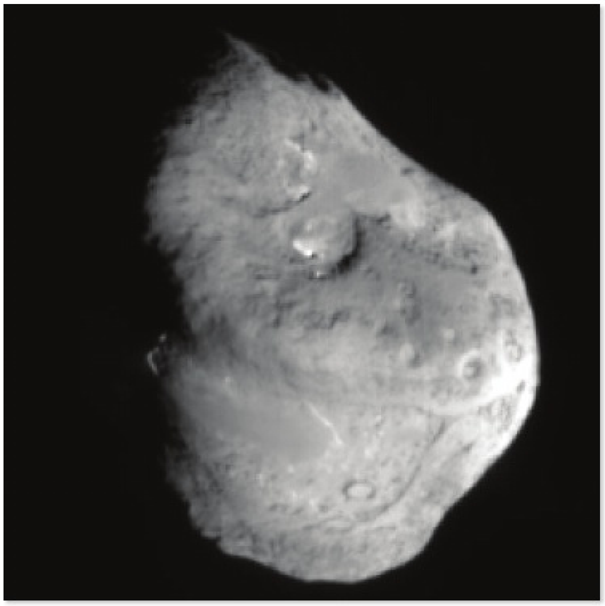


fig D The comet Tempel 1 was hit by NASA's Deep Impact probe.

On 4 July 2005, NASA's Deep Impact mission succeeded in crashing a spacecraft into a comet called Tempel 1 (**fig D**). For that mission, the impactor spacecraft had a mass of 370 kg compared with the comet's mass of 7.2×10^{13} kg, so there would have been an insignificant change in the comet's trajectory. Deep Impact was purely intended to study the comet's composition. However, there is an asteroid named Apophis which has a small chance of colliding with Earth in 2035, 2036, or maybe 2037, and there have been some calls for a mission to crash a spacecraft into Apophis in order to deviate it out of harm's way. The mass of this asteroid is 6.1×10^{10} kg and it is travelling at $12.6\,\text{km}\,\text{s}^{-1}$. It has been claimed that a collision by a 4000 kg impactor craft travelling at $6\,\text{km}\,\text{s}^{-1}$ could alter the path of this asteroid enough to ensure it would not hit Earth. If this impactor collided with Apophis at right angles, we can calculate the change in angle of the asteroid (**fig E**).

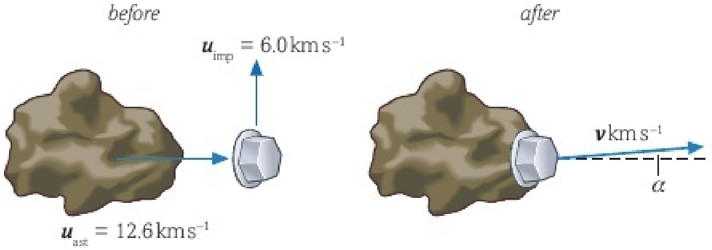


fig E Could we hit an asteroid hard enough to save Earth from Asteroid Impact Hazards?

Before collision:

$$p_{ast} = m_{ast}u_{ast} = 6.1 \times 10^{10} \times 12.6 \times 10^{3} = 7.69 \times 10^{14}\,\text{kg}\,\text{m}\,\text{s}^{-1}$$
$$p_{imp} = m_{imp}u_{imp} = 4 \times 10^{3} \times 6 \times 10^{3} = 2.4 \times 10^{7}\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

fig F The vector sum of momentum components after asteroid impact.

The momentum of the combined object after the impactor embeds in the asteroid is the vector sum of the two initial momenta, which are at right angles to each other.

After collision:

$$p_{after} = \sqrt{(p_{ast}{}^{2} + p_{imp}{}^{2})} = \sqrt{((7.69 \times 10^{14})^{2} + (2.4 \times 10^{7})^{2})}$$
$$= 7.69 \times 10^{14}\,\text{kg}\,\text{m}\,\text{s}^{-1}$$
$$\therefore \quad v_{after} = \frac{p_{after}}{m_{total}} = \frac{7.69 \times 10^{14}}{(6.1 \times 10^{10} + 4 \times 10^{3})}$$
$$= 12.6\,\text{km}\,\text{s}^{-1} \text{ (3 significant figures)}$$

There is no significant change in the magnitude of the asteroid's velocity – what about its direction?

Angle of momentum after:

$$\alpha = \tan^{-1}\left(\frac{2.4 \times 10^{7}}{7.69 \times 10^{14}}\right) = 1.79 \times 10^{-6\,\circ}$$

Although less than two microdegrees sounds like an insignificantly small angle, this would represent a change in position of nearly 30 km as Apophis crosses the Earth's orbit from one side of the Sun to the other. This might be just enough to avert a collision with Earth that would have a hundred times more energy than all the explosives used in the Second World War.

Investigation

Investigating 2D collisions

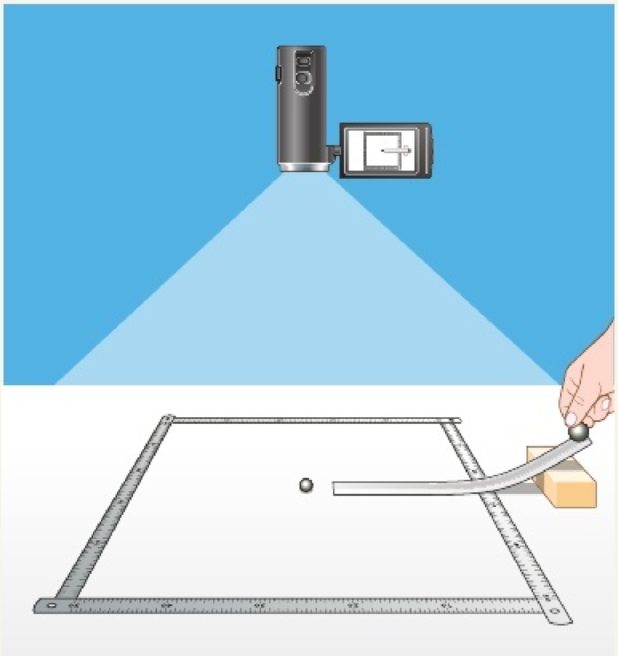


fig G Video analysis of collisions in 2D.

You can investigate two-dimensional collisions in the school laboratory. We saw in **Book 1, Section 2.1.2** that by analysing video footage of an object's movement, frame by frame, we can calculate any changes in velocity. With measurement scales in two dimensions, the components of velocity in each dimension can be isolated. Thus, separate calculations can be made in each dimension, in order to verify the conservation of momentum in 2D. Make sure you have a good understanding of this practical as your understanding of the experimental method may be assessed in your examinations.

Questions

1 (a) What is the impulse needed to stop a car that has a momentum of 22 000 kg m s^{-1}?

(b) If the car brakes could apply a force of 6800 N, how long would it take to bring the car to a stop?

2 In a pool shot, the cue ball has a mass of 0.17 kg. It travels at 6.00 m s^{-1} and hits the stationary black ball in the middle of one end of the table. The black ball, also of mass 0.17 kg, travels away at 45° and 4.24 m s^{-1}, ending up in the corner pocket.

(a) By resolving the components of the black ball's momentum find out what happens to the cue ball.

(b) Is this an elastic or inelastic collision?

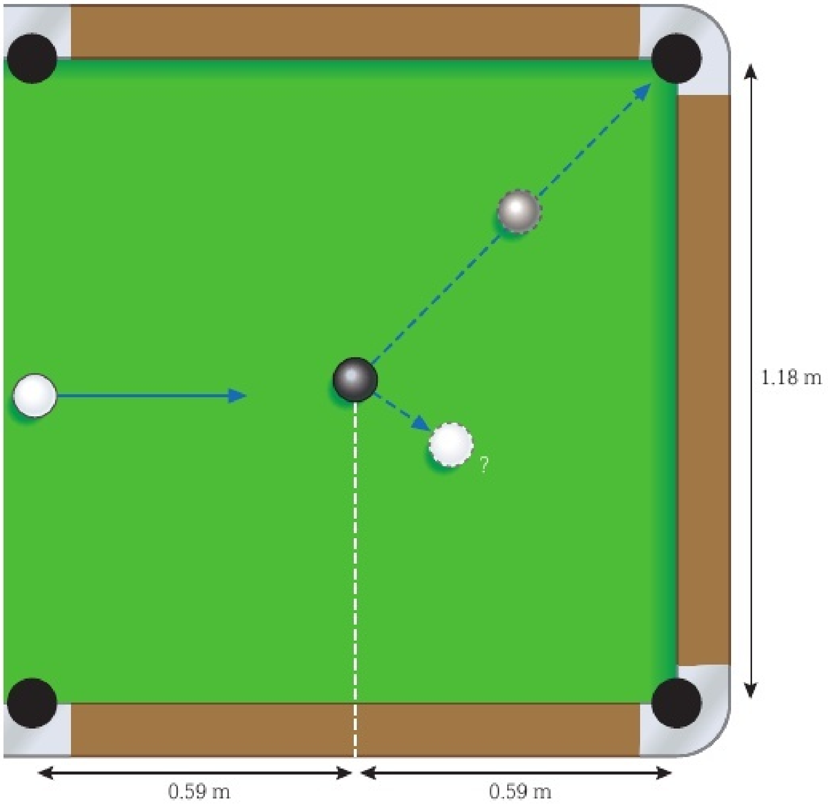


fig H 2D momentum calculations can be very important in winning at pool.

3 Calculate how fast the impactor spacecraft in **fig E** would have to be travelling if it is to alter the Apophis asteroid's trajectory by one degree. Comment on the answer.

4 For the experimental set up shown in **fig G**, suggest two improvements that could be made in order to improve the accuracy of the resolved vectors that would be observable on the video stills.

Key definition

The **impulse** is force acting for a certain time causing a change in an object's momentum.

impulse = $F \times \Delta t$

THINKING BIGGER

END OF THE WORLD?

The 1998 science fiction movie, *Armageddon*, starring Bruce Willis, was released within two months of the movie *Deep Impact* starring Robert Duvall, also science fiction.

fig A *Armageddon*, starring Bruce Willis, was a box office rival to *Deep Impact* with Robert Duvall.

Armageddon

A huge comet is headed for Earth. As it does not consist of one big piece, but of a cloud of rather small pieces plus a main rock the size of Texas, little impacts are recorded long before the big one is scheduled for collision with Earth. Harry S. Stamper and his team of misfit drillers are selected to land on the main comet as they are the only ones who can work the special drill Harry developed. Their mission is to drill 800 ft into the comet to place a nuclear explosive device. The explosion of the bomb will break the comet in two, and the two pieces will pass Earth on both sides. This task has to be accomplished before a certain ‘dead line’, or the comet parts will not fly by, but hit Earth.

- Extract from the IMDb website, a user posted plot summary by Julian Reischl.

Deep Impact

Leo Biederman (Elijah Wood) discovers a new comet at his school astronomy club. When the discovery is submitted to an astronomer, he races to warn people about it, but is killed in a car crash. A reporter, Jenny Lerner (Tea Leoni) is investigating an affair by a member of the government. He mysteriously resigned, claiming he wanted to spend time with his family. Jenny confronts him and discovers the secret of the comet – it is on a collision course with the Earth. She is summoned to see President Tom Beck, by black SUVs crashing into her car. The President (Morgan Freeman) asks her to keep the prospect of this Extinction Level Event a secret. Jenny insists that it must be made public, and she forces a press conference. There is confusion in Leo’s family, and home town, as the President claims he was the discovering astronomer and that he died a year earlier. President Beck explains that there is now a mission called Messiah to intercept the comet and destroy it. The alternative, should the mission fail, is for a selection of one million people to be made, who will take cover underground and continue humanity in the future. People over 50 years old will be excluded from the lottery for a place in the caves. Jenny has a number of family and personal travails to deal with alongside her reporting of the Messiah mission. As the astronauts, led by Captain Tanner (Robert Duvall) say farewells to their families and set off, many people wonder if the mission will succeed, and what their futures may hold.

- From the notes of an anonymous movie reviewer.

Where else will I encounter these themes?

Book 1 | 6.1 | YOU ARE HERE | 6.2 | 7.1 | 7.2 | 7.3 | 8.1

Let us start by considering the nature of the writing in the two plot summaries.

1. The extracts were written by members of the public.

   a. Discuss the tone, level of vocabulary and level of scientific detail that has been used – who is the intended audience?

   b. Discuss any differences in narrative approach between the two movies that are evident from the information given in each plot summary.

2. Explain which of the two movies sounds the most appealing to you as a viewer, including what aspects of the movie appeal most to you, based solely on the plot summaries above.

Consider whether the storytelling in the movies focuses on technology and scientific points, human relationships, everyday events or high stakes plot points, everyday people or high level leaders.

Now we will look at the physics in detail. Some of these questions will link to topics elsewhere in this book, so you may need to combine concepts from different areas of physics to work out the answers.

3. a. Explain the physics behind the plan in *Armageddon* to ensure the main rock does not collide with Earth.

   b. In *Armaggedon*, explain the existence of the 'dead line' after which parts of the comet would still hit the Earth even if it were broken up by the explosion.

4. Texas is roughly 1000 km across. *Armageddon*'s comet was depicted with an asteroid-like structure of iron and rock. The overall density of such asteroids is about $2000\,\text{kg m}^{-3}$.

   a. Calculate the depth that the nuclear bomb is to be placed in this comet as a percentage of its size. Comment on the plan to break up the comet this way. (1 foot = 30 cm)

   b. Estimate the volume of the comet in *Armageddon*, and in this way calculate your estimate for the comet's mass.

   c. The movie claims this comet is travelling at $10\,000\,\text{m s}^{-1}$ and the bomb must split the comet no less than four hours before impact. The Earth's radius is 6400 km. Calculate the force that the nuclear explosion, lasting for 2 seconds, would need to apply to send the two parts off course enough to save the Earth.

5. An Extinction Level Event (*Deep Impact* plot summary) is one in which a significant quantity and diversity of the life on Earth becomes extinct. There have been several such events in history. Comets are typically tens of kilometres in diameter. Whilst a comet impact of this size would cause great destruction locally, it could not directly kill a significant amount of the life on Earth. Do some research and explain how a comet impact could lead to an Extinction Level Event.

Think about what would determine the motion of the broken pieces after the explosion – consider conservation of momentum.

Activity

Write a plot summary for a movie you have seen which has a significant scientific or technological element to the plot or setting. Use the extracts to guide you in terms of the structure and length of your own summary.

It is important to make sure that you include all aspects of the movie, not just the plot, the characters, or the setting. At the same time, make sure that your writing is as concise as possible.

6.1 Exam-style questions

[Note: In questions marked with an asterisk (*), marks will be awarded for your ability to structure your answer logically showing how the points that you make are related or follow on from each other.]

1 An inelastic collision:

A conserves momentum but not kinetic energy

B conserves momentum and kinetic energy

C need not conserve energy

D need not conserve momentum. [1]

[Total: 1]

2 A tennis ball travelling with the momentum of 4.2 kg m s^{-1} is hit by a tennis racquet. The force of 56 N from the racquet causes the tennis ball to travel back in the opposite direction with 5.8 kg m s^{-1}. How long is the ball in contact with the racquet?

A 0.029 s

B 0.10 s

C 0.18 s

D 5.6 s [1]

[Total: 1]

3 In order to calculate the kinetic energy of a non-relativistic particle, we would need to know its:

A mass only

B mass and momentum

C acceleration and momentum

D velocity and acceleration. [1]

[Total: 1]

4 A spacecraft called Deep Space 1, mass 486 kg, uses an 'ion-drive' engine. This type of engine is designed to be used in deep space.

The following statement appeared in a website.

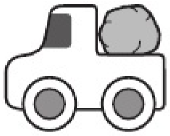
> The ion propulsion system on Deep Space 1 expels 0.13 kg of xenon propellant each day. The xenon ions are expelled from the spacecraft at a speed of 30 km s^{-1}. The speed of the spacecraft is predicted to initially increase by about 8 m s^{-1} each day.

Use a calculation to comment on the prediction made in this statement. [4]

[Total: 4]

5 (a) Explain what is meant by the principle of conservation of momentum. [2]

(b) The picture shows a toy car initially at rest with a piece of modelling clay attached to it.



A student carries out an experiment to find the speed of a pellet fired from an air rifle. The pellet is fired horizontally into the modelling clay. The pellet remains in the modelling clay as the car moves forward. The motion of the car is filmed for analysis.

The car travels a distance of 69 cm before coming to rest after a time of 1.3 s.

(i) Show that the speed of the car immediately after being struck by the pellet was about 1 m s^{-1}. [2]

(ii) State an assumption you made in order to apply the equation you used. [1]

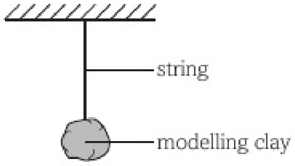
(iii) Show that the speed of the pellet just before it collides with the car is about 120 m s^{-1}
mass of car and modelling clay = 97.31 g
mass of pellet = 0.84 g [3]

(c) The modelling clay is removed and is replaced by a metal plate of the same mass. The metal plate is fixed to the back of the car. The experiment is repeated but this time the pellet bounces backwards.

*(i) Explain why the speed of the toy car will now be greater than in the original experiment. [3]

(ii) The film of this experiment shows that the pellet bounces back at an angle of 72° to the horizontal. Explain why the car would move even faster if the pellet bounced directly backwards at the same speed. [1]

(d) The student tests the result of the first experiment by firing a pellet into a pendulum with a bob made of modelling clay. They calculate the energy transferred.



The student's data and calculations are shown:

Data:

mass of pellet = 0.84 g

mass of pendulum and pellet = 71.6 g

change in vertical height of pendulum = 22.6 cm

Calculations:

change in gravitational potential energy of pendulum and pellet

$= 71.6 \times 10^{-3}\,\text{kg} \times 9.81\,\text{N kg}^{-1} \times 0.226\,\text{m} = 0.16\,\text{J}$

therefore kinetic energy of pendulum and pellet immediately after collision = 0.16 J

therefore kinetic energy of pellet immediately before collision = 0.16 J

therefore speed of pellet before collision = $19.5\,\text{m s}^{-1}$

There are no mathematical errors but the student's answer for the speed is too small.

Explain which of the statements in the calculations are correct and which are not. [4]

[Total: 16]

6 James Chadwick is credited with 'discovering' the neutron in 1932.

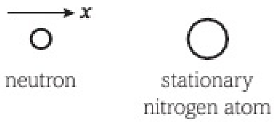
Beryllium was bombarded with alpha particles, knocking neutrons out of the beryllium atoms. Chadwick placed various targets between the beryllium and a detector. Hydrogen and nitrogen atoms were knocked out of the targets by the neutrons and the kinetic energies of these atoms were measured by the detector.

(a) The maximum energy of a nitrogen atom was found to be 1.2 MeV.

Show that the maximum velocity of the atom is about $4 \times 10^6\,\text{m s}^{-1}$.

mass of nitrogen atom = $14u$, where $u = 1.66 \times 10^{-27}\,\text{kg}$ [3]

(b) The mass of a neutron is Nu (where N is the relative mass of the neutron) and its initial velocity is $\boldsymbol{x}$. The nitrogen atom, mass $14u$, is initially stationary and is then knocked out of the target with a velocity, $\boldsymbol{y}$, by a collision with a neutron.



(i) Show that the velocity, $\boldsymbol{z}$, of the neutron after the collision can be written as

$$\boldsymbol{z} = \frac{\mathrm{N}\boldsymbol{x} - 14\boldsymbol{y}}{\mathrm{N}}$$ [3]

(ii) The collision between this neutron and the nitrogen atom is elastic. What is meant by an elastic collision? [1]

(iii) Explain why the kinetic energy E_k of the nitrogen atom is given by

$$E_\mathrm{k} = \frac{\mathrm{N}u(\boldsymbol{x}^2 - \boldsymbol{z}^2)}{2}$$ [2]

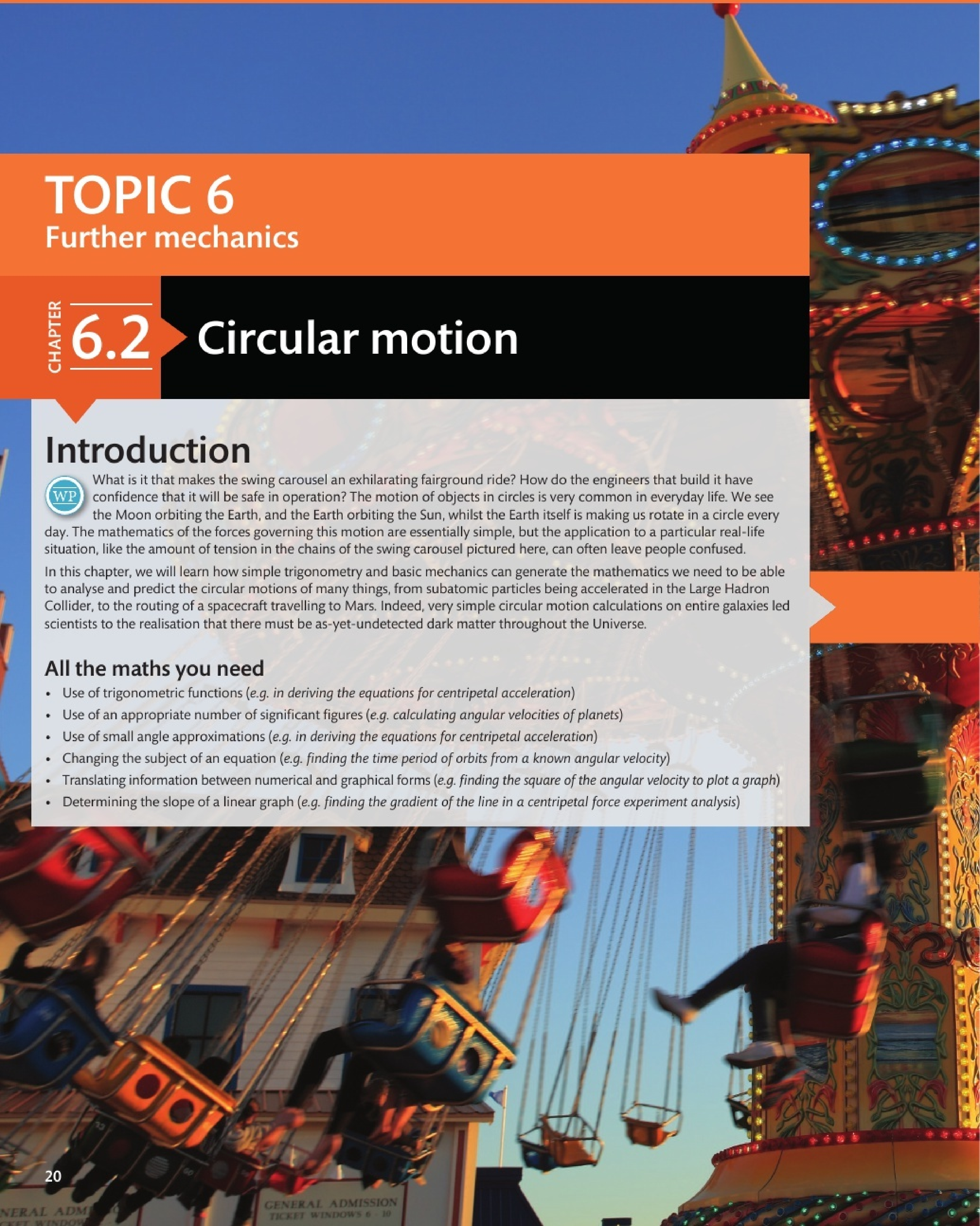
(c) The two equations in (b) can be combined and $\boldsymbol{z}$ can be eliminated to give

$$\boldsymbol{y} = \frac{2\mathrm{N}\boldsymbol{x}}{\mathrm{N} + 14}$$

(i) The maximum velocity of hydrogen atoms knocked out by neutrons in the same experiment was $3.0 \times 10^7\,\text{m s}^{-1}$. The mass of a hydrogen atom is $1u$. Show that the relative mass N of the neutron is 1. [3]

(ii) This equation can not be applied to all collisions in this experiment. Explain why. [1]

[Total: 13]



TOPIC 6
Further mechanics

CHAPTER
6.2 Circular motion

Introduction

What is it that makes the swing carousel an exhilarating fairground ride? How do the engineers that build it have confidence that it will be safe in operation? The motion of objects in circles is very common in everyday life. We see the Moon orbiting the Earth, and the Earth orbiting the Sun, whilst the Earth itself is making us rotate in a circle every day. The mathematics of the forces governing this motion are essentially simple, but the application to a particular real-life situation, like the amount of tension in the chains of the swing carousel pictured here, can often leave people confused.

In this chapter, we will learn how simple trigonometry and basic mechanics can generate the mathematics we need to be able to analyse and predict the circular motions of many things, from subatomic particles being accelerated in the Large Hadron Collider, to the routing of a spacecraft travelling to Mars. Indeed, very simple circular motion calculations on entire galaxies led scientists to the realisation that there must be as-yet-undetected dark matter throughout the Universe.

All the maths you need

- Use of trigonometric functions (*e.g. in deriving the equations for centripetal acceleration*)
- Use of an appropriate number of significant figures (*e.g. calculating angular velocities of planets*)
- Use of small angle approximations (*e.g. in deriving the equations for centripetal acceleration*)
- Changing the subject of an equation (*e.g. finding the time period of orbits from a known angular velocity*)
- Translating information between numerical and graphical forms (*e.g. finding the square of the angular velocity to plot a graph*)
- Determining the slope of a linear graph (*e.g. finding the gradient of the line in a centripetal force experiment analysis*)

What have I studied before?

- Speed and acceleration
- How to add vectors
- How to resolve vectors
- Newton's laws of motion
- The radian
- Tension forces

What will I study later?

- The circular motion of charged particles in magnetic fields
- Particle accelerators
- Particle detectors
- The Large Hadron Collider
- Gravitational fields and orbital motion
- Simple harmonic motion

What will I study in this chapter?

- How to use both radians and degrees in angle measurements
- The concept of angular velocity, and how to calculate it
- Centripetal acceleration and how to derive its equations
- The need for a centripetal force to enable circular motion
- Calculations of centripetal force

6.2 1 Circular motion basics

By the end of this section, you should be able to...

- express angular displacement in radians and in degrees, and convert between these units
- define *angular velocity*, and make calculations using it
- define *centripetal acceleration*, and derive and use the equations for it

In our study of wave phase in **Book 1, Section 5.2.1**, we learned how angles can be measured in radians as well as in degrees, and how to convert between these two units. In this section, we will see how that can be taken further to measure movements around a circle.

Going round in circles

For an object moving in a circle, we often need to measure where it is around that circle. For example, to consider the relative positions of planets in their orbits at a particular moment, we need to be able to state where each one is (ignoring, for the sake of this example, the fact that planetary orbits are not perfect circles!).

Angles measured in degrees are used extensively in navigation in order to locate places and discuss the difference between moving from one starting point to two possible destinations. This is measuring angular displacement on the surface of the Earth. Each degree is subdivided into 60 'minutes' and each of those minutes into 60 'seconds'.

fig A Measuring angles in degrees.

When we are measuring rotation, we often use the alternative unit to measure angles – the **radian**. This is defined by the nature of a circle itself. Imagine an object moves around part of the circumference of a circle. The angle through which it moves, measured in radians, is defined as the distance it travels, divided by its distance from the centre of the circle (the radius). If the radius of the circle were one metre, then the distance the object travels around the circumference (also in metres) would be equal to the angle swept out in radians.

$$\text{angle (in radians)} = \frac{\text{length of arc}}{\text{radius of arc}}$$

$$\theta = \frac{s}{r}$$

So, for a complete circle, in which the circumference is equal to $2\pi r$, the angle swept out would be:

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

This means that the angle will be 1 radian (rad) if the distance moved around the circle is the same as the radius; just over $\frac{1}{6}$ of the distance around the circumference.

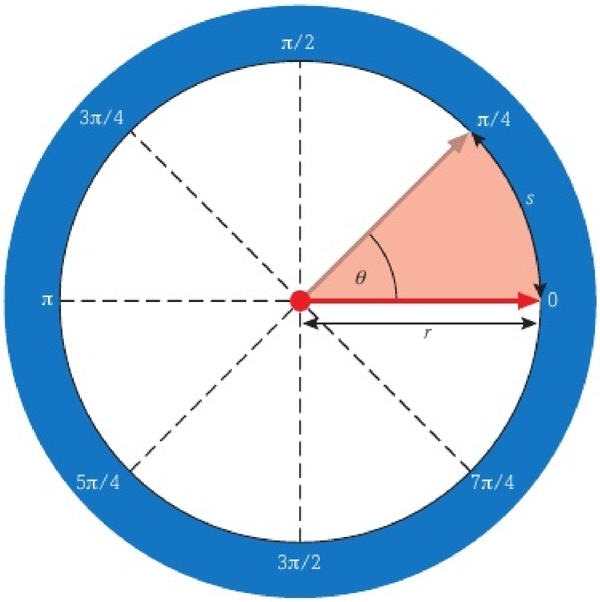


fig B Measuring angles in radians.

Angular displacement is the vector measurement of the angle through which something has moved. The standard convention is that anticlockwise rotation is a positive number and clockwise rotation is a negative number.

Angle in radians	Angle in degrees
0	0
π/4	45
π/2	90
3π/4	135
π	180
5π/4	225
3π/2	270
7π/4	315
2π	360

table A Angles measured in degrees and radians.

Did you know?

Military angles

The British Army uses a system for angle measurement in which a complete circle is divided into 6400 'mils'. This is an abbreviation for milliradian. They have rounded the numbers for easy use though, as strictly speaking there should be 6283 milliradians in a complete circle. The idea is that at 1 kilometre distance, an angle of 1 mil would represent a distance of 1 metre. So, when aiming artillery fire, a horizontal adjustment of 1 mil in angle of fire should move to a target 1 metre sideways for every kilometre distance away.

$$s = r\theta$$

$\therefore$ for 1 mil angle at 1 km distance:

$$s = 1000 \times 0.001 = 1\text{ m}$$

fig C Measuring angles in mils.

Angular velocity

An object moving in a circle sweeps out a certain angle in a certain time, depending upon how fast it is moving. The rate at which the angular displacement changes is called the **angular velocity,** ω. So, angular velocity is measured in rad s^{-1}, and is defined mathematically by:

$$\omega = \frac{\theta}{t}$$

If the object completes a full circle (2π radians) in a time period, T, then the angular velocity is given by:

$$\omega = \frac{2\pi}{T}$$

$$\therefore \quad T = \frac{2\pi}{\omega}$$

The frequency of rotation is the reciprocal of the time period.

$$f = \frac{1}{T}$$

$$\therefore \quad \omega = 2\pi f$$

Instantaneous velocity

Rather than thinking about the angular movement, let us consider the actual velocity through space of the moving object (sometimes called the 'instantaneous velocity'). We know that $v = \frac{s}{t}$ and from the definition of the angle in radians $\theta = \frac{s}{r}$, so that $s = r\theta$.

Thus: $v = \frac{r\theta}{t}$

$$v = r\omega$$

WORKED EXAMPLE

In **fig D** we can see a geostationary satellite orbiting the Earth. What is its angular velocity?

In order to find the angular velocity, remember that it completes an orbit at the same rate as the Earth revolves, so one full circle every 24 hours.

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{(24 \times 60 \times 60)} = \frac{2\pi}{86\,400}$$

$$\omega = 7.27 \times 10^{-5}\text{ rad s}^{-1}$$

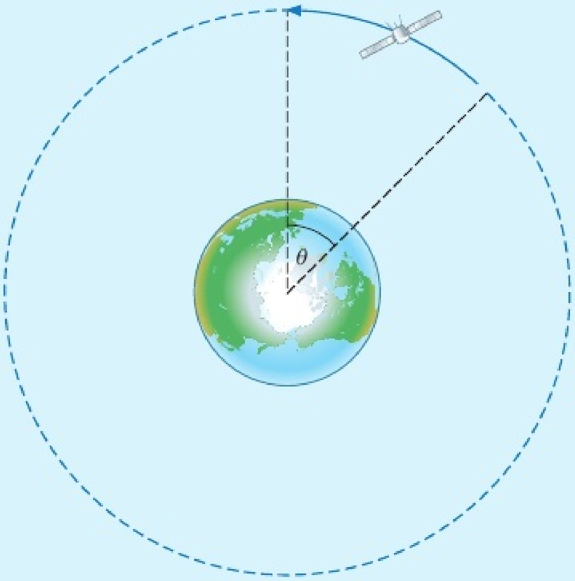


fig D How quickly does a satellite rotate through a certain angle?

If the radius of the Earth is 6400 km and the satellite in **fig D** is in orbit 35 600 km above the Earth's surface, what is the velocity of the satellite?

From before, $\omega = 7.27 \times 10^{-5}\text{ rad s}^{-1}$

$$v = r\omega = (6400 + 35\,600) \times 10^3 \times 7.27 \times 10^{-5}$$

$$\therefore \quad v = 3050\text{ m s}^{-1}$$

Centripetal acceleration

Velocity is a vector, and so it is correctly described by quoting both its magnitude and direction. An acceleration can change either of these, or both. An object moving in a circle may travel at a constant speed (and a constant angular velocity) but the direction it is moving in must constantly change. This means it is constantly accelerating. As this acceleration represents the changes in direction around the circle, it is called the **centripetal acceleration, *a*.** In order to determine how to calculate the centripetal acceleration, we must consider how quickly the direction, and therefore the velocity, is changing.

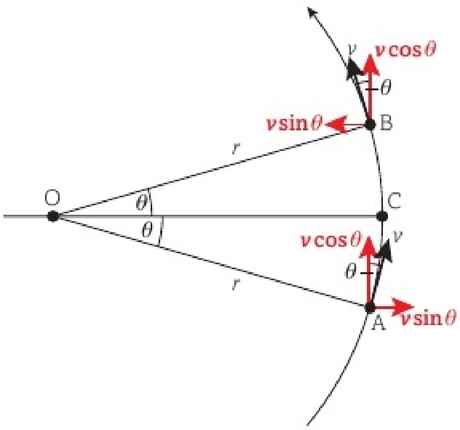


fig E Vector components of velocity leads us to the centripetal acceleration equation.

At the arbitrary positions of the rotating object, A and, at a time t later, B, we consider the components of the object's velocity, in the x and y directions.

As A and B are equal distances above and below point C, the vertical velocity component, v_y, is the same in both cases:

$$v_y = v\cos\theta$$

So the vertical acceleration is zero:

$$a_y = 0$$

Horizontally, the magnitude of the velocity is equal at both points, but in opposite directions:

at A: $v_x = v\sin\theta$

at B: $v_x = -v\sin\theta$

So, the acceleration is just the horizontal acceleration, calculated as change in velocity divided by time:

$$a_x = \frac{2v\sin\theta}{t}$$

From the definition of angular velocity above:

$$v = \frac{r\theta}{t}$$

$$\therefore \quad t = \frac{r\theta}{v}$$

Here, the angle moved in time t is labelled as 2θ, so:

$$t = \frac{r2\theta}{v}$$

$$\therefore \quad a_x = \frac{v2v\sin\theta}{r2\theta} = \frac{v^2}{r}\frac{\sin\theta}{\theta}$$

This must be true for all values of θ, and as we want to find the instantaneous acceleration at any point on the circumference, we must consider the general answer as we reduce θ to zero. In the limit, as θ tends to zero:

$$\frac{\sin\theta}{\theta} = 1$$

$$\therefore \quad \text{centripetal acceleration } a = \frac{v^2}{r}$$

From the definition of the instantaneous velocity above:

$$v = r\omega$$

$$\therefore \quad a = \frac{(r\omega)^2}{r}$$

$$\therefore \quad \text{centripetal acceleration } a = r\omega^2$$

The centripetal acceleration in this case is just the horizontal acceleration, as we considered the object in a position along a horizontal radius. Following a similar derivation at any point around the circle will always have identical components of velocity that are perpendicular to the radius on either side of the point being considered. Thus, the centripetal acceleration is always directed towards the centre of the circle.

WORKED EXAMPLE

What is the centripetal acceleration of the satellite in **fig D**?

$$a = \frac{v^2}{r} = \frac{(3050)^2}{(6400 + 35\,600) \times 10^3}$$

$$a = 0.22\ \text{m s}^{-2}$$

or:

$$a = r\omega^2 = (6400 + 35\,600) \times 10^3 \times (7.27 \times 10^{-5})^2$$

$$a = 0.22\ \text{m s}^{-2}$$

Questions

1 Convert:

(a) 4π radians into degrees

(b) 36° into radians.

2 What is the angular velocity of an athletics hammer if the athlete spins at a rate of three revolutions per second?

3 Vinyl records are played at one of three speeds. Calculate the angular velocity of each:

(a) (i) 33 revolutions per minute

(ii) 45 revolutions per minute

(iii) 78 revolutions per minute.

(b) Vinyl records played at the speeds in part (a) (i) are usually 12 inches (or 30 cm) in diameter. What would be the centripetal acceleration of a point on the outside circumference of a record such as this?

4 A man standing on the Equator will be moving due to the rotation of the Earth.

(a) What is his angular velocity?

(b) What is his instantaneous velocity?

(c) What is his centripetal acceleration?

5 What is the percentage error in the British Army measurement system that uses 6400 mils for a complete circle?

Key definitions

A **radian** is a unit of angle measurement, equivalent to 57.3 degrees.

Angular displacement is the vector measurement of the angle through which something has moved.

Angular velocity ω is the rate at which the angular displacement changes, unit, radians per second.

Centripetal acceleration, a is the acceleration towards the centre of a circle that corresponds to the changes in direction to maintain an object's motion around that circle.

6.2 2 Centripetal force

By the end of this section, you should be able to...

- explain that a centripetal force is required to produce and maintain circular motion
- use the equations for centripetal force

In **Section 6.2.1** we saw how an object moving in a circle must be constantly accelerating towards the centre of the circle in order to maintain its motion around the circle.

Why circular motion?

When a hammer thrower whirls an athletics hammer around in a circle, the hammer has an angular velocity. Yet when the thrower lets the hammer go, it will fly off, following a straight line which is the direction in which it was moving at the instant of release. This direction is always along the edge of the circle (a tangent) at the point when it was released.

fig A The instantaneous velocity of an object moving in a circle is tangential to the circle. When there is no resultant force, velocity will be constant, so it moves in a straight line.

As the hammer is whirled at a constant speed, the magnitude of the velocity is always the same. However, the direction of the velocity is constantly changing. This means that the vector of velocity is constantly changing, and a change in velocity is an acceleration. Newton's first law tells us that acceleration will only happen if there is a resultant force. The hammer is constantly being pulled towards the centre of the circle. In this example, the force providing this pull is the tension in the string (or chain). For any object moving in a circle, there must be a resultant force to cause this acceleration, and it is called the **centripetal force**.

Learning tip

There is no special centripetal force, any resultant force that makes an object move in a circle is labelled as the centripetal force for that object and circle. For example, a satellite will move in a circular orbit around the Earth because its weight acts towards the centre of the Earth. The weight is the only force acting, so could be referred to as the centripetal force.

Centripetal force

fig B Astronauts are subject to extreme acceleration forces. These forces are simulated in training by the centripetal force in a giant centrifuge.

The mathematical formula for the centripetal force on an object moving in a circle can be found from Newton's second law, and the equation we already have for the centripetal acceleration:

$$F = ma \quad \text{and} \quad a = \frac{v^2}{r}$$

$$F = \frac{mv^2}{r}$$

$$\text{centripetal force} = \frac{\text{mass} \times (\text{velocity})^2}{\text{radius}}$$

Noting that $v = r\omega$, there is an alternative equation for centripetal force in terms of angular velocity:

$$F = \frac{mv^2}{r} = \frac{m(r\omega)^2}{r}$$

$$F = mr\omega^2$$

The resultant centripetal force needed will be larger if:

- the rotating object has more mass
- the object rotates faster
- the object is further away from the centre of the circle.

WORKED EXAMPLE

Estimate the centripetal force on an astronaut in the astronaut training centrifuge (see **fig B**) if his capsule rotated once every 2 seconds.

Estimate of radius of revolution: $r = 6$ m.

Estimate of astronaut's mass: $m = 80$ kg.

$$\text{velocity, } v = \frac{s}{t} = \frac{2\pi(6)}{2} = 18.8\text{ m s}^{-1}$$

$$\text{centripetal force, } F = \frac{mv^2}{r} = \frac{(80) \times (18.8)^2}{6} = 4700\text{ N}$$

This is about 6 times his weight.

If the operator of the centrifuge were to increase its rate of rotation to once every second, what would the astronaut's angular velocity, centripetal force and acceleration now be?

$$\text{angular velocity, } \omega = 2\pi f = 6.28\text{ rad s}^{-1}$$

$$\text{centripetal force, } F = mr\omega^2 = (80) \times (6) \times (6.28)^2 = 18\,900\text{ N}$$

$$\text{centripetal acceleration, } a = r\omega^2 = (6) \times (6.28)^2 = 237\text{ m s}^{-2}$$

This is about 24 times the acceleration due to gravity and would most likely be fatal if maintained for more than a few seconds.

Learning tip

Circular motion equations summary:

$$\text{angular displacement, } \theta = \frac{s}{r}$$

$$\text{angular velocity, } \omega = 2\pi f = \frac{2\pi}{T} = \frac{v}{r}$$

$$\text{centripetal acceleration, } a = r\omega^2 = \frac{v^2}{r}$$

$$\text{centripetal force, } F = mr\omega^2 = \frac{mv^2}{r}$$

Investigation

Investigating centripetal force

fig C Verifying the centripetal force equation. Eye protection should be worn during this investigation.

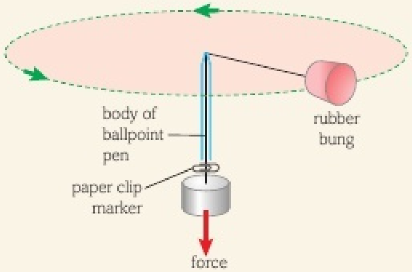


fig D Experimental detail for centripetal force experiment.

You can investigate the centripetal force equation by spinning a rubber bung on a string around in a circle. The tension in the string, which is the centripetal force, will be provided by the hanging masses at the bottom of the vertical string and thus will be known. Spin the rubber bung around in a circle at a speed that keeps a paperclip marker in a constant position near the handle. The paperclip marker allows you to maintain a fixed length of string (radius) which you can measure. You will also need to measure the mass of the rubber bung. Your partner can then time ten revolutions in order to give you the angular velocity. Take angular velocity measurements for different forces (different numbers of hanging masses).

$$F = mr\omega^2$$

$$\therefore \quad \omega^2 = \frac{F}{mr}$$

A graph of ω^2 plotted against F should give a straight best-fit line. The gradient of this line will be $\frac{1}{mr}$

Questions

1 A roller coaster has a complete (circular) loop with a radius of 20 m. A 65 kg woman rides the roller coaster and the car travels once round the loop in 4.5 seconds. What centripetal force does the woman experience?

2 A man with a mass of 75 kg standing on the Equator is moving because of the rotation of the Earth.

(a) What centripetal force is required to keep him moving in this circle?

(b) How does this compare with his weight?

(c) How would the reaction force with the ground be different if he went to the South Pole? (Assume the Earth is a perfect sphere.)

Key definition

Centripetal force is the resultant force towards the centre of the circle to maintain an object's circular motion.

THINKING BIGGER

ARTIFICIAL GRAVITY

Astronauts on long-term space missions would benefit from the generation of artificial gravity within their ships. Common designs for this involve rotating the spaceship, but this movement can lead to motion sickness.

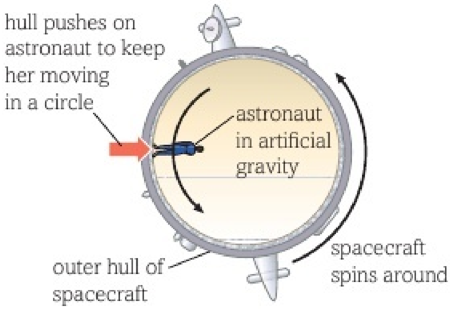


fig A This is how artificial gravity is generated in a spacecraft.

Artificial gravity, as it is usually conceived, is the inertial reaction to the centripetal acceleration that acts on a body in circular motion. Artificial-gravity environments are often characterized in terms of four parameters:

- **radius** from the centre of rotation (in metres)
- **angular velocity** or 'spin rate' (usually quoted in revolutions per minute)
- **tangential velocity** or 'rim speed' (in metres per second)
- **centripetal acceleration** or 'gravity level' (in multiples of the Earth's surface gravity).

Comfort criteria

Deliberate architectural design for the unusual conditions of artificial gravity ought to aid adaptation and improve the habitability of the environment (Hall).

Here is a summary of five research articles which consider the parameters leading to rotational discomfort:

Author	Year	Radius [m]	Angular Velocity [rpm]	Tangential Velocity [m/s]	Centripetal Acceleration [g]	
		min.	max.	min.	min.	max.
Hill & Schnitzer	1962	not known	4	6	0.035	1.0
Gilruth	1969	12	6	not known	0.3	0.9
Gordon & Gervais	1969	12	6	7	0.2	1.0
Stone	1973	4	6	10	0.2	1.0
Cramer	1985	not known	3	7	0.1	1.0

Radius Because centripetal acceleration – the nominal artificial gravity – is directly proportional to radius, inhabitants will experience a head-to-foot 'gravity gradient'. To minimize the gradient, maximize the radius.

Angular velocity The cross-coupling of normal head rotations with the habitat rotation can lead to dizziness and motion sickness. To minimize this cross-coupling, minimize the habitat's angular velocity.

Graybiel conducted a series of experiments in a 15-foot-diameter 'slow rotation room' and observed:

> *In brief, at 1.0 rpm even highly susceptible subjects were symptom-free, or nearly so. At 3.0 rpm subjects experienced symptoms but were not significantly handicapped. At 5.4 rpm, only subjects with low susceptibility performed well and by the second day were almost free from symptoms. At 10 rpm, however, adaptation presented a challenging but interesting problem. Even pilots without a history of air sickness did not fully adapt in a period of twelve days.*

On the other hand, Lackner and DiZio found that:

> *Sensory-motor adaptation to 10 rpm can be achieved relatively easily and quickly if subjects make the same movement repeatedly. This repetition allows the nervous system to gauge how the Coriolis forces generated by movements in a rotating reference frame are deflecting movement paths and endpoints, and to institute corrective adaptations.*

Tangential velocity When people or objects move within a rotating habitat, they're subjected to Coriolis accelerations that distort the apparent gravity. For relative motion in the plane of rotation, the ratio of Coriolis to centripetal acceleration is twice the ratio of the relative velocity to the habitat's tangential velocity. To minimize this ratio, maximize the habitat's tangential velocity.

Centripetal acceleration The centripetal acceleration must have some minimum value to offer any practical advantage over weightlessness. One common criterion is to provide adequate floor traction. The minimum required to preserve health remains unknown.

● From a webpage to calculate artificial gravity, maintained by Ted Hall: Hall, Theodore W. (2012). 'SpinCalc: An Artificial-Gravity Calculator in JavaScript'; www.artificial-gravity.com/sw/SpinCalc/SpinCalc.htm (as at 24 October 2014).

Where else will I encounter these themes?
Book 1 | 6.1 | 6.2 | YOU ARE HERE | 7.1 | 7.2 | 7.3 | 8.1

Let us start by considering the nature of the writing on the webpage.

1. The extract above consists of information from an American space architect. Consider the extract and comment on the type of writing that is used in each case. Try and answer the following questions.

a. Discuss the tone and level of vocabulary included in the article – who is the intended audience?

b. Discuss the level of scientific detail included in the article, particularly considering the intended audience.

c. Compare the webpage of the extract with this more recent website http://spacearchitect.org/ considering style and presentation, in addition to content.

The main basis for the website is a calculator that calculates the artificial gravity for various different sizes and speeds of rotation. You may find it helpful to visit the website to view it and its list of references.

Now we will look at the physics in detail. Some of these questions will link to topics elsewhere in this book, so you may need to combine concepts from different areas of physics to work out the answers.

2. a. Explain the author's suggestion that 'artificial gravity ... is proportional to radius'.

b. What would a 'gravity gradient' be? Explain how it would come about and why it would be minimised by maximising the radius.

3. a. Convert the angular velocity in Gilruth's data into SI units.

b. Calculate the centripetal force on an 82 kg astronaut using Gilruth's data.

c. If this astronaut tried to walk by providing a tangential force of 350 N, calculate the moment caused by his foot, acting on the floor about the spacecraft's central axis.

4. Do some research on the Coriolis force effect, and use your research to explain the final sentence in the section on angular velocity.

5. Explain how a spaceship, in space, could be made to start rotating.

Considering the equations for centripetal acceleration, remember that the tangential velocity is dependent on radius.

Activity

Imagine that Mr Hall, the space architect, has been invited to give a talk at your school about artificial gravity. You have been asked to prepare some demonstrations to illustrate how the data in the experiments referenced above could have been measured. Prepare instructions for two demonstrations you will do.

You will need to consider how to highlight where and how measurements would be made.

6.2 Exam-style questions

[Note: In questions marked with an asterisk (*), marks will be awarded for your ability to structure your answer logically showing how the points that you make are related or follow on from each other.]

1 Which of the following is **not** a correct unit for angular velocity?

A rad min^{-1}

B degrees per minute

C rad s

D $^{\circ}$ s^{-1} [1]

[Total: 1]

2 Considering the centripetal force to make a car drive around a roundabout, it is more likely to skid outwards if:

A it has fewer passengers

B it has worn tyres

C it travels more slowly

D it drives around further from the centre of the roundabout. [1]

[Total: 1]

3 A particle moves in a circle, completing 14.5 complete revolutions in one minute. What is its angular velocity?

A 0.24 rad s^{-1}

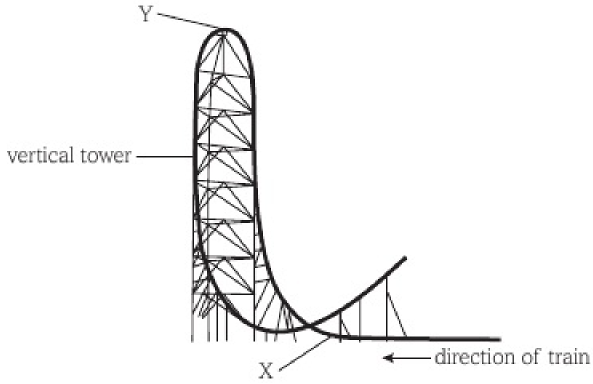
B 1.52 rad s^{-1}

C 14.5 rad s^{-1}

D 87 rad s^{-1} [1]

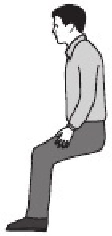
[Total: 1]

4 Kingda Ka was the highest roller coaster in the world in 2007. A train is initially propelled along a horizontal track by a hydraulic system. It reaches a speed of 57 m s^{-1} from rest in 3.5 s. It then climbs a vertical tower before falling back towards the ground.



(a) Calculate the average force used to accelerate a fully loaded train along the horizontal track.
Total mass of fully loaded train = 12 000 kg [2]

(b) Point X is just before the train leaves the horizontal track and moves into the first bend. Complete the free-body diagram below to show the two forces acting on a rider in the train at this point. [3]



(c) The mass of the rider is m and g is the acceleration of free fall. Just after point X, the reaction force of the train on the rider is 4 mg and can be assumed to be vertical.
This is referred to as a g-force of 4 g.
Show that the radius of curvature of the track at this point is about 100 m. [3]

(d) Show that the speed of the train as it reaches the top of the vertical tower is about 20 m s^{-1}. Assume that resistance forces are negligible.
The height of the vertical tower is 139 m. [2]

(e) Riders will feel momentarily weightless if the vertical reaction force becomes zero.
The track is designed so that this happens at point Y.
Calculate the radius of the track at point Y. [2]

[Total: 12]

5 Astronauts can be weakened by the long-term effects of microgravity. To keep in shape it has been suggested that they can do some exercise using a Space Cycle: a horizontal beam from which an exercise bike and a cage are suspended. One astronaut sits on the exercise bike and pedals, which causes the whole Space Cycle to rotate around a pole. Another astronaut standing in the cage experiences artificial gravity. When rotated at 20 revolutions per minute, this is of similar strength to the gravitational field on Earth.